



Catchup exam of Operational research

Duration: 1h

Session: May 2021

EXERCISE 1: (2 + 2 + 2 + 2 = 8 Marks)

1. Using the simplex procedure, solve the following linear program on the right.

$$\begin{aligned} & \text{maximize} && -x_1 + x_2 \\ & \text{subject to} && x_1 - x_2 \leq 2 \\ & && x_1 + x_2 \leq 6 \\ & && x_1 \geq 0, \quad x_2 \geq 0. \end{aligned}$$

2. Draw a graphical representation of the problem (question 1) in x_1, x_2 space and indicate with an arrow, the path of the simplex steps.
3. Consider the constraint set in E^2 defined in terms of the inequalities, in the right. Draw the feasible set according to the constraint set.

$$\begin{aligned} & x_1 + \frac{8}{3}x_2 \leq 4 \\ & x_1 + x_2 \leq 2 \\ & 2x_1 \leq 3 \\ & x_1 \geq 0, \quad x_2 \geq 0. \end{aligned}$$

4. Is the feasible set of question 3, convex? If yes list the extreme points, else justify your answer

EXERCISE 2: (1 + 2 + 2 + 2 = 7 Marks)

Let us consider the following tableau produced by an iteration of the simplex method using the vector interpretation and solving the linear program *minimize* $c^T x$ *subject to* $Ax = b$ and $x \geq 0$; where x is an n -dimensional column vector, c^T is an n -dimensional row vector, A is an $m \times n$ matrix, and b is an m -dimensional column vector. The vector inequality $x \geq 0$ means that each component of x is nonnegative. Based on the tableau, answer to the questions below

a_1	a_2	a_3	\dots	a_m	a_{m+1}	a_{m+2}	\dots	a_n	b
1	0	0	\dots	0	$\bar{a}_{1(m+1)}$	$\bar{a}_{1(m+2)}$	\dots	\bar{a}_{1n}	\bar{a}_{10}
0	1	0	\dots	0	$\bar{a}_{2(m+1)}$	$\bar{a}_{2(m+2)}$	\dots	.	\bar{a}_{20}
0	0	1	\dots	.	.	.	\dots	.	.
.	.	.	\dots	.	.	.	\dots	.	.
.	.	.	\dots	.	.	.	\dots	.	.
0	0	0	\dots	1	$\bar{a}_{m(m+1)}$	$\bar{a}_{m(m+2)}$	\dots	\bar{a}_{mn}	\bar{a}_{m0}

1. Is the tableau in canonical form?
2. List the a_j belonging to the basis
3. What is the current value of the solution vector x
4. Write the vector a_{m+2} as linear combination of basic vectors of question 2

EXERCISE 3: (5 Marks)

Solve the following linear program

$$\begin{aligned} & \text{minimize} && -2x_1 + 4x_2 + 7x_3 + x_4 + 5x_5 \\ & \text{subject to} && -x_1 + x_2 + 2x_3 + x_4 + 2x_5 = 7 \\ & && -x_1 + 2x_2 + 3x_3 + x_4 + x_5 = 6 \\ & && -x_1 + x_2 + x_3 + 2x_4 + x_5 = 4 \\ & && x_2 \geq 0, \quad x_3 \geq 0, \quad x_4 \geq 0, \quad x_5 \geq 0. \end{aligned}$$

Good Luck